

# Frustrated quantum antiferromagnetism with ultracold bosons in a triangular lattice

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**Abstract.** - We propose to realize the anisotropic triangular-lattice Bose-Hubbard model with positive tunneling matrix elements by using ultracold atoms in an optical lattice dressed by a fast lattice oscillation. This model exhibits frustrated antiferromagnetism at experimentally feasible temperatures; it interpolates between a classical rotor model for weak interaction, and a quantum spin-1/2  $XY$ -model in the limit of hard-core bosons. This allows to explore experimentally gapped spin liquid phases predicted recently [Schmied et al., New J. Phys. **10**, 045017 (2008)].

**Introduction.** – Frustrated quantum antiferromagnetism can give rise to extraordinarily rich physics [1]. It is not only supposed to play a crucial role for the properties of high- $T_c$  superconductors, but it is also an interesting subject on its own, and has potential applications in topological quantum information processing and storage [2]. Apart from the possibility of keeping a classical Néel-ordered (staggered or spiral) spin configuration with long-range order, the spins of a quantum antiferromagnet can also form singlet pairs (“valence bonds”), such that the spin-rotation symmetry is not broken. These singlets either order spatially to form a valence bond solid or the system’s state is a superposition of many singlet coverings, neither breaking translational nor spin-rotational symmetry. The latter is termed a resonating valence bond spin liquid (SL). A gapped SL with an exponential decay of spin correlations is expected to exhibit non-local topological order being immune against local perturbations. It also can possess anyonic excitations. This makes such *topological* SL states candidates for robust quantum memories and processors [2]. Alternatively, a *critical* SL is characterized by a huge density of low-lying excitations and a power-law decay of spin correlations.

Since frustrated quantum antiferromagnets are hard to simulate (path-integral Monte-Carlo methods fail) and

clean solid-state realizations are not available, it is desirable to study these exotic many-body systems with ultracold atoms in optical lattice potentials [3] providing both clean conditions and far-reaching control. The most straightforward cold atom implementation of a quantum magnet is to create a Mott-insulator of fermions in two different internal states forming a pseudo spin. However, the necessary low temperatures (smaller than the weak superexchange spin coupling) have not yet been achieved. Also spinless fermions at filling 2/3 in a not yet realized trimerized Kagomé lattice resemble a quantum magnet [4].

In this letter we propose a different strategy for the realization of a frustrated quantum system with ultracold atoms that — in contrast to the aforementioned approaches — can be pursued in existing experimental setups, at temperatures already reached. Our idea is to consider spinless ultracold bosonic atoms in a triangular optical lattice and to induce frustration via a sign change of the matrix elements describing tunneling between adjacent potential minima. As will be shown, such a sign change can be achieved effectively by dressing the system with a high-frequency elliptical lattice acceleration. In the hard-core boson limit of strong repulsive interaction, the physics is then described by the antiferromagnetic spin-1/2  $XY$ -model on the triangular lattice. For certain regimes of anisotropic coupling this model is expected to show gapped SL phases [5].

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This letter is organized as follows. We start with a discussion of the frustrated positive-hopping Bose-Hubbard model that describes the system to be realized experimentally. In order to sketch the expected phase diagram we combine (i) recently published numerical data (based on PEPS as well as exact simulations) [5] valid in the limit of strong interaction with (ii) results obtained by starting from the limit of weak interaction and systematically including quantum fluctuations (beyond Bogoliubov). Then we discuss the experimental realization of the model, putting emphasis on how to change the sign of the tunneling matrix elements via a fast elliptical lattice acceleration. It follows a part devoted to the preparation of the frustrated model's ground state in the presence of a trapping potential. Finally, before giving a brief conclusion, we discuss possible experimental signatures of the expected phases.

**Positive-hopping Bose-Hubbard model on a triangular lattice.** — Consider a sample of ultracold bosonic atoms in a deep triangular optical lattice that is forced inertially by moving the lattice rapidly along an elliptical orbit. According to the following section, the time evolution of such a system has a simple description. Integrating out the fast oscillatory motion on the short time scale  $T = 2\pi/\omega$  of the elliptical forcing, one finds the system's evolution on longer time scales governed by the time-independent effective Bose-Hubbard Hamiltonian

$$\hat{H}_{\text{eff}} = \sum_{\langle ij \rangle} J_{ij}^{\text{eff}} \hat{b}_i^\dagger \hat{b}_j + \sum_i \left[ \frac{U}{2} \hat{n}_i (\hat{n}_i - 1) - \mu_i \hat{n}_i \right]. \quad (1)$$

Here  $\hat{b}_i$  and  $\hat{n}_i$  are the bosonic annihilation and number operators for Wannier states localized at the minima  $\mathbf{r}_i$  of the triangular lattice potential. The first term comprises tunneling between adjacent sites  $i$  and  $j$  with — this is the crucial point — matrix elements  $J_{ij}^{\text{eff}}$  that are smoothly tunable from negative to positive values by variation of the forcing strength.<sup>1</sup> The on-site terms are characterized by the positive interaction parameter  $U$  and the local chemical potential  $\mu_i \equiv \mu - V_i$  including the trapping potential  $V_i$ . We consider the anisotropic lattice shown in Fig. 1(a) with the  $J_{ij}^{\text{eff}}$  equal to either  $J$  or  $J' \equiv \alpha J$  (assuming  $\alpha \geq 0$ ).

The homogeneous model ( $\mu_i = \mu$ ) interpolates between a classical rotor and a quantum spin model: For weak interaction  $U \ll n|J|$ , with a mean filling of  $n$  particles per site, the superfluid (SF) ground state can (locally) be approximated by  $\prod_i \exp(\psi_i \hat{b}_i^\dagger) |\text{vacuum}\rangle$  with discrete order parameter  $\psi_i = \sqrt{n_i} \exp(i\varphi_i)$ . A homogeneous density  $n_i = n$  is favored and the local phases  $\varphi_i$  play the role of classical rotors assuming a configuration  $\varphi_i \equiv \mathbf{q} \cdot \mathbf{r}_i$  described by the ordering vector  $\mathbf{q}$ . Antiferromagnetic coupling  $J > 0$  implies Néel ordered phases  $\varphi_i$  as depicted in Fig. 1(b-e). We call such a state a Néel SF. When

<sup>1</sup>In our convention  $\langle ij \rangle$  denotes an oriented pair of neighboring sites,  $\langle ij \rangle \neq \langle ji \rangle$ .

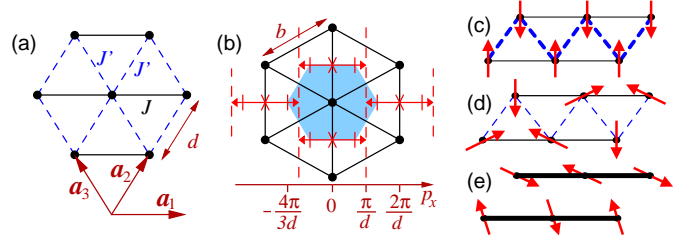


Fig. 1: (color online) (a) Anisotropic triangular lattice considered, with primitive vectors  $\mathbf{a}_1 \equiv d\mathbf{e}_x$ ,  $\mathbf{a}_2 \equiv d[(1/2)\mathbf{e}_x + (\sqrt{3}/2)\mathbf{e}_y]$ , as well as  $\mathbf{a}_3 \equiv -\mathbf{a}_1 + \mathbf{a}_2$ . The tunneling matrix elements  $J_{ij}^{\text{eff}}$  take values  $J$  and  $J' \equiv \alpha J$  (with  $\alpha \geq 0$ ) along the solid and dashed bonds, respectively. (b) Reciprocal lattice with  $b = (4\pi/\sqrt{3})d^{-1}$ . The first Brillouin-zone, centered at  $\mathbf{p} = \mathbf{0}$ , is shaded. Considering antiferromagnetic coupling  $J > 0$ , we have marked the ordering vector  $\mathbf{q}$  describing a Néel SF in the limit of weak interaction: For  $\alpha \geq \alpha_0$   $\mathbf{q}$  lies on one of the x-shaped crosses (being equivalent modulo reciprocal lattice vectors). This corresponds to a staggered configuration of the local phase angles  $\varphi_i$  on the rhombic lattice of  $J'$ -bonds [shown in (c) with the  $\varphi_i$  visualized by pointers]. Lowering  $\alpha$ , at  $\alpha = \alpha_0$  the position of  $\mathbf{q}$  splits in a continuous way into two non-equivalent possible positions that separate symmetrically along the arrows drawn in (b). The phases  $\varphi_i$  assume a spiral pattern with two possible chiralities; subfigure (d) corresponds to the isotropic lattice ( $\alpha = 1 < \alpha_0$ ) with  $\mathbf{q}$  lying on one of the corners of the first Brillouin zone. Finally, in the 1D limit ( $\alpha = 0$ ) only  $q_x$  has a defined value that is marked by the dashed lines in (b). The phase pattern is staggered along the 1D chains of  $J$ -bonds [as sketched in (e)].

$\alpha$  exceeds a value  $\alpha_0$ , spiral continuously transforms into staggered Néel order. While  $\alpha_0$  equals 2 for  $U/J = 0$ , it slightly decreases with increasing interaction, cf. Fig. 3(a).

In the opposite limit of strong interaction  $U \gg n|J|$ , there are only two energetically favored site occupations,  $n_i = [n] \equiv g$  (the largest integer smaller than  $n$ ) and  $n_i = g + 1$ . Associating them with “spin up” and “spin down”, respectively, gives the Bloch-sphere representation  $|\vartheta_i, \varphi_i\rangle \equiv \cos(\vartheta_i/2)|g\rangle_i + \sin(\vartheta_i/2)\exp(i\varphi_i)|g+1\rangle_i$  at each site. Replacing  $(g+1)^{-1}\hat{b}_i$  by the spin lowering operator  $(\hat{\sigma}_i^x - i\hat{\sigma}_i^y)/2$ , one arrives at the XY-model

$$\hat{H}_{XY} = \sum_{\langle ij \rangle} J_{ij}^{XY} (\hat{\sigma}_i^x \hat{\sigma}_j^x + \hat{\sigma}_i^y \hat{\sigma}_j^y) + \sum_i h_i \hat{\sigma}_i^z \quad (2)$$

with  $h_i \equiv \frac{1}{2}(\mu_i - Ug)$ ,  $J_{ij}^{XY} \equiv \frac{g+1}{4} J_{ij}^{\text{eff}}$ , and  $\hat{\sigma}_i^x, \hat{\sigma}_i^y, \hat{\sigma}_i^z$  being spin-1/2 Pauli operators at site  $i$ . The ground state of  $\hat{H}_{XY}$  cannot be a product state like  $\prod_i |\vartheta_i, \varphi_i\rangle$  with definite local phases  $\varphi_i$  anymore, since  $|\vartheta_i, \varphi_i\rangle$  cannot be an eigenstate of both  $\hat{\sigma}_i^x$  and  $\hat{\sigma}_i^y$ . Viewed from the Bose-Hubbard perspective, increasing interparticle repulsion increases the fluctuations of the local phases  $\varphi_i$ . While for ferromagnetic coupling  $J < 0$  the classical phase configuration is supposed to survive the presence of quantum fluctuations in the spin-1/2 limit  $U \gg n|J|$ , for antiferromagnetic coupling  $J > 0$  recent simulations suggest that (for  $\sum_i \langle \hat{\sigma}_i^z \rangle = 0$ ) classical Néel order is not necessarily

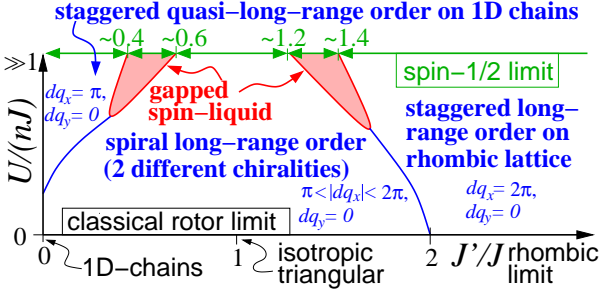


Fig. 2: (color online) Sketch of the phase diagram of the anisotropic positive-hopping Bose-Hubbard model on the triangular lattice for half-odd-integer filling. The parameter plane is spanned by interaction strength  $U/(nJ)$  and anisotropy ratio  $\alpha = J'/J$ . The data (in green) for the spin-1/2 limit [ $U/(nJ) \gg 1$ ] are taken from Ref. [5]. We assume the SL phases to survive at small finite values of  $J/U$ , since they are protected by a gap. The behaviour at small  $U/(nJ)$  corresponds to results obtained within a generalized Bogoliubov theory, cf. Fig. 3(a).

preserved [5]: Along the  $\alpha$ -axis different Néel phases are separated by gapped SL phases with exponentially decaying spin correlations. The results of Ref. [5] are displayed along the upper edge of the phase diagram shown in Fig. 2.

In order to gain further insight into the physics of the frustrated positive-hopping Bose-Hubbard model (1), we start from the classical limit of weak interaction  $U \ll n|J|$  (assuming a homogeneous system) and include quantum fluctuations by using the generalized Bogoliubov approach introduced in Ref. [6]. For filling already moderately larger than 1, we can replace  $\hat{b}_i \simeq \exp[i(\varphi_i + \delta\varphi_i)]\sqrt{n_i + \delta\hat{n}_i}$ , where  $\delta\hat{n}_i \simeq \delta\hat{n}_i^\dagger$  and  $\delta\hat{\varphi}_i \simeq \delta\hat{\varphi}_i^\dagger$  describe quantum fluctuations to the local particle numbers  $n_i$  and phases  $\varphi_i$ , respectively, with  $[\delta\hat{n}_i, \delta\hat{\varphi}_j] \simeq i\delta_{i,j}$ . While  $\langle(\delta\hat{n}_i)^2\rangle_i/n^2 \ll 1$  can be assumed, the phase fluctuations  $\langle(\delta\hat{\varphi}_i)^2\rangle$  diverge in the 1D limit ( $\alpha = 0$ ) (as well as at finite temperatures) where only quasi-long-range order is possible. However, one can still expect the fluctuation of the *relative* phases  $\langle(\delta\hat{\varphi}_i - \delta\hat{\varphi}_j)^2\rangle$  between neighboring sites  $i$  and  $j$  to be small. Expanding the Hamiltonian (1) up to second order in  $\delta\hat{n}_i/n_i$  and  $(\delta\hat{\varphi}_i - \delta\hat{\varphi}_j)$ , it will be quadratic in terms of new bosonic operators  $\hat{d}_i \equiv \sqrt{n_i}[\delta\hat{n}_i/(2n_i) + i\delta\hat{\varphi}_i]$  and  $\hat{d}_i^\dagger$ , and can be diagonalized by a Bogoliubov transform (keeping  $\langle\delta\hat{n}_i\rangle = 0$ ). When computing, e.g., correlations  $\langle\hat{b}_i^\dagger\hat{b}_j\rangle$  between distant sites  $i$  and  $j$ , one cannot treat  $(\delta\hat{\varphi}_i - \delta\hat{\varphi}_j)$  as a small quantity, but has to use Wick's theorem to evaluate expectation values of all powers of  $\delta\hat{\varphi}_i$  [6]. We augment this analysis by taking into account also the (Wick-decomposed) quartic corrections to the Hamiltonian when minimizing the ground-state energy with respect to both the Bogoliubov coefficients and the ordering vector  $\mathbf{q}$ .<sup>2</sup> This self-consistent above-Bogoliubov correction, that we include using a numeric iteration scheme, is necessary in

<sup>2</sup>In the case of spiral order ( $\pi < d|q_x| < 2\pi$  with  $q_y = 0$ ), one finds two solutions,  $\mathbf{q}$  and  $\mathbf{q}' = -\mathbf{q}$ , and has to choose one of them.

order to explain a shift of  $\alpha_0$  with increasing interaction.<sup>3</sup>

Assuming homogeneous filling  $n_i = n$  as well as  $q_y = 0$ , the method sketched in the above paragraph leads to the following results: With increasing interaction/quantum fluctuations,  $\alpha_0$  decreases, i.e. the  $\alpha$ -domain of rhombic-staggered Néel order grows [thick line in Fig. 3(a)]. This “order by disorder” phenomenon [1] is in accordance with the spin-1/2 results of Ref. [5] (cf. upper edge of Fig. 2). In contrast, a finite  $\alpha$ -interval of staggered 1D quasi-long-range Néel order, also predicted for the spin model, is not found. We have used these findings to draw the lower part of the phase diagram of Fig. 2. The quasiparticle dispersion relation is gapless and phonon-like for quasimomentum wave numbers  $\mathbf{p}$  around  $\mathbf{p} = \mathbf{q}$ . However, whenever spiral order is found, it is symmetric with respect to  $(\mathbf{p} - \mathbf{q}) \rightarrow -(\mathbf{p} - \mathbf{q})$  only in the limit of small  $|\mathbf{p} - \mathbf{q}|$ . In contrast, the zero temperature quasimomentum distribution, having sharp peaks at  $\mathbf{p} = \mathbf{q} +$  reciprocal lattice vectors, possesses reflection symmetry with respect to  $\mathbf{p} = \mathbf{q}$ . Further results, for  $n = 3.5$ , are shown in Fig. 3(a). An estimate for the range of validity of the approximation is given by the dotted and the dashed line. Above them the relative phase fluctuations between neighboring sites separated by  $\mathbf{a}_1$  and  $\mathbf{a}_2$ , respectively, exceed a value taken to be  $\pi/4$ . The fact that the dotted line does not approach zero in the limit of decoupled 1D chains ( $\alpha \rightarrow 0$ ) indicates that in this limit the approximation still captures the physics in  $\mathbf{a}_1$ -direction (along the chains). Moreover, the dip around  $\alpha = \alpha_0$  can be interpreted as a precursor of the SL phase predicted in Ref. [5] (cf. Fig. 2). Finally, the thin solid line marks the interaction where the condensate fraction  $f_c \equiv \lim_{|\mathbf{r}_{ij}| \rightarrow \infty} |\langle\hat{b}_i^\dagger\hat{b}_j\rangle|/n$  is reduced to 0.75. Again, a sharp dip at  $\alpha = \alpha_0$  is a hint at a quantum disordered phase in the limit of large interaction.

**Proposal for an experimental realization.** – Having discussed the physics of the triangular-lattice positive-hopping Bose-Hubbard Hamiltonian, let us turn to the realization of the model with ultracold atoms in a deep optical lattice. The sign change of the tunneling matrix element, from negative to positive values, shall be induced by dressing the system with a fast time-periodic lattice acceleration. For hypercubic lattices, such a dynamical modification of tunneling has been predicted theoretically not only for single [8], but also for many interacting particles [9]. Moreover, it has been observed experimentally with ultracold atoms both in the weakly interacting regime (via the expansion of a Bose-Einstein condensate [10]), as well as in the strong-coupling regime, where it has been used to induce the quantum phase transition from a SF to a Mott insulator (MI) and back [9, 11]. However, the linear driving scheme used in the work just mentioned, with the

<sup>3</sup>Taking into account self-consistently the quartic terms does not lead to a spurious gap in the quasiparticle spectrum, as it is the case within the standard Bogoliubov treatment [7]. This gaplessness is, thus, a feature of the generalized Bogoliubov expansion [6] in terms of fluctuations in density and relative-phase.

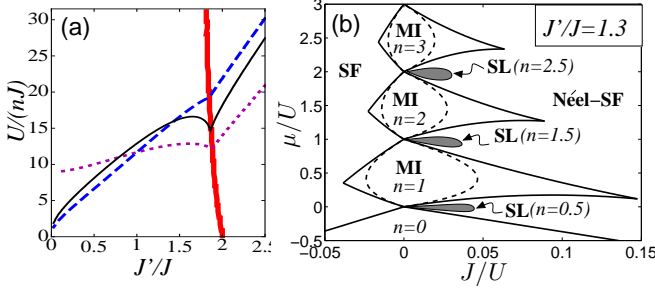


Fig. 3: (color online) (a) Generalized Bogoliubov theory for  $n = 3.5$ . Values of  $U/(n|J|)$  at which: spiral changes to rhombic-staggered Néel order (thick line), relative phase fluctuations reach  $\pi/4$  for sites separated by  $\mathbf{a}_1$  (dotted line) and  $\mathbf{a}_2$  (dashed line), the condensate fraction has dropped to 0.75 (thin solid line). (b) Boundaries of the MI phases with integer filling  $n$  in the  $\mu/U$ - $J/U$ -plane, both in 2nd-order strong-coupling (solid lines) and meanfield (dashed lines) approximation. As a consequence of frustration, the MI double-lobes are larger on the antiferromagnetic side ( $J > 0$ ) of the phase diagram. The grey bubbles between the MI regions, indicating the expected gapped SL phases at half-odd-integer filling, are just sketched.

system being forced sinusoidally along a single direction (chosen to be diagonal with respect to all symmetry axes in the case of a square or a cubic lattice), is not suitable for the triangular lattice geometry. In order to be able to manipulate the system in a symmetric way with respect to the three non-orthogonal lattice directions  $\mathbf{a}_1$ ,  $\mathbf{a}_2$ , and  $\mathbf{a}_3$  [Fig. 1(a)], here we propose to use elliptical forcing. This includes isotropic circular as well as linear forcing.

The driving scheme to be considered can be realized inertially by moving the lattice along an elliptical orbit  $\mathbf{x}(t) = \Delta x_c \cos(\omega t) \mathbf{e}_c + \Delta x_s \sin(\omega t) \mathbf{e}_s$  in space, with angular frequency  $\omega$ , orthogonal unit vectors  $\mathbf{e}_c$  and  $\mathbf{e}_s$ , as well as amplitudes  $\Delta x_c$  and  $\Delta x_s$ . The resulting inertial force in the lattice frame of reference reads  $\mathbf{F}(t) = -m\ddot{\mathbf{x}} = F_c \cos(\omega t) \mathbf{e}_c + F_s \sin(\omega t) \mathbf{e}_s$  where  $m$  is the boson mass and  $F_{c/s} = m\omega^2 \Delta x_{c/s}$ . Choosing  $\omega$  and  $F_{c/s}$  small enough to exclude transitions from the lowest to higher Bloch bands, one can describe the system in the lattice frame of reference by the driven Bose-Hubbard model

$$\hat{H}(t) = \sum_{\langle ij \rangle} J_{ij} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + \sum_i [v_i(t) - \mu_i] \hat{n}_i. \quad (3)$$

Here  $J_{ij} < 0$  are the bare tunneling matrix elements and  $v_i(t) \equiv -\mathbf{r}_i \cdot \mathbf{F}(t)$  oscillating on-site energies.

We assume that  $\hbar\omega$  is large compared to the energy scales given by interaction ( $U$ ), tunneling ( $n|J_{ij}|$ ), and trapping ( $|\mu_i - \mu_j|$ , with neighbors  $i$  and  $j$ ), i.e. that the forcing is fast with respect to the time scales governing the undriven model. Under these conditions the time evolution of the driven system's state  $|\psi(t)\rangle$  will be to good approximation of the form

$$|\psi(t)\rangle \approx \hat{U}(t) |\psi_{\text{eff}}(t)\rangle. \quad (4)$$

The unitary operator

$$\hat{U}(t) \equiv \exp \left( -\frac{i}{\hbar} \sum_i \hat{n}_i W_i(t) \right), \quad (5)$$

where

$$W_i(t) \equiv \int_0^t d\tau v_i(\tau) - \frac{1}{T} \int_0^T dt' \int_0^{t'} d\tau v_i(\tau), \quad (6)$$

just describes a periodically time-dependent shift by  $-m\dot{\mathbf{x}}$  of the whole system in quasimomentum,  $W_i = \mathbf{r}_i \cdot m\dot{\mathbf{x}}$ . On top of this simple oscillatory motion on the short time scale  $T = 2\pi/\omega$ , the time evolution on longer times is governed by the effective time-independent Hamiltonian  $\hat{H}_{\text{eff}}$  shown in Eq. (1), namely

$$|\psi_{\text{eff}}(t)\rangle = \exp \left( -\frac{i}{\hbar} \hat{H}_{\text{eff}} t \right) |\psi_{\text{eff}}(0)\rangle. \quad (7)$$

The dressed tunneling matrix elements are given by

$$J_{ij}^{\text{eff}} = J_{ij} J_0 \left( \frac{K_{ij}}{\hbar\omega} \right). \quad (8)$$

Here  $J_0$  is the zero-order Bessel function and  $K_{ij} \equiv \sqrt{(F_c \mathbf{e}_c \cdot \mathbf{r}_{ij})^2 + (F_s \mathbf{e}_s \cdot \mathbf{r}_{ij})^2}$  the amplitude of the potential modulation between site  $i$  and  $j$ , where  $\mathbf{r}_{ij} \equiv \mathbf{r}_i - \mathbf{r}_j$ . Thus, in the lattice frame, apart from the superimposed fast oscillation in quasimomentum, the system behaves as the one described by  $\hat{H}_{\text{eff}}$ . When measuring the momentum distribution of the system in the laboratory frame by taking time-of-flight absorption images, one will encounter the periodic quasimomentum distribution of  $|\psi_{\text{eff}}\rangle$  at rest, being enveloped by the momentum distribution of the Wannier wave function oscillating like  $m\dot{\mathbf{x}}$ .

The result presented in the preceding paragraph relies on the separation of time scales as well as on time averaging. We have obtained it within the framework of quantum Floquet theory [12] by generalizing the approach introduced in Refs. [9,13] to elliptical forcing. The derivation is based on stationary degenerate-state perturbation theory on the level of an extended Hilbert space including time as a coordinate. Here we just give a simple argument making the  $\hat{H}_{\text{eff}}$ -description plausible: Transforming  $|\psi'\rangle = \hat{U}^\dagger |\psi\rangle$  leads to the new Hamiltonian  $\hat{H}' = \hat{U}^\dagger \hat{H} \hat{U} - i\hbar \hat{U}^\dagger (d_t \hat{U})$ . Accordingly,  $\hat{H}'$  is obtained from  $\hat{H}$  by subtracting the oscillating potential terms  $\propto v_i(t)$  and replacing  $J_{ij} \rightarrow J_{ij} \exp(i[W_i - W_j]/\hbar)$ . Now the rapidly oscillating phase factors in the tunneling terms of  $\hat{H}'$  can approximately be taken into account on time average,  $\hat{H}'(t) \rightarrow \frac{1}{T} \int_0^T dt \hat{H}'(t) = \hat{H}_{\text{eff}}$ , giving  $|\psi'\rangle \approx |\psi_{\text{eff}}\rangle$ .

A 2D triangular optical lattice can be realized by superimposing three laser beams, all polarized in  $z$ -direction, at an angle of  $2\pi/3$  in the  $xy$ -plane, while a standing light wave in  $z$ -direction is used to create a stack of effectively two-dimensional systems. A further beam in  $z$ -direction allows to modify the trapping potential in the  $xy$ -plane.



The lattice motion can be realized by varying the relative frequencies of the beams by means of acousto-optical modulators. For the purposes described above, an orbit  $\mathbf{x}(t) = \Delta x_c \cos(\omega t) \mathbf{e}_c + \Delta x_s \sin(\omega t) \mathbf{e}_s$  is required, with  $\Delta x_{s/c}$  on the order of a lattice constant and  $\omega/(2\pi)$  being a few kHz. Starting from an isotropic undriven lattice with bare tunneling matrix elements  $J_{ij} = \bar{J} < 0$  and choosing  $\mathbf{e}_{c/s} = \mathbf{e}_{x/y}$ , one obtains effective tunneling matrix elements (8) distributed as depicted in Fig. 1(a). Namely  $K_{ij}$  reads  $K \equiv d|F_c|$  and  $K' \equiv d\sqrt{F_c^2 + 3F_s^2}/2$  along the solid and dashed bonds, respectively, giving  $J = \bar{J}J_0(K/(\hbar\omega))$  and  $J' = \bar{J}J_0(K'/(\hbar\omega))$  according to Eq. (8). This allows for any value of the anisotropy parameter  $\alpha = J'/J$ .

We have already implemented a triangular optical lattice in the laboratory, loaded it with ultracold  $^{87}\text{Rb}$  atoms, and observed the transition from a SF to a MI. Also a controlled motion of the lattice has been achieved.

**State preparation and role of trapping potential.** – For elliptical forcing there are no instants in time where  $\hat{U}(t)$  is equal to the identity (i.e. with  $\dot{\mathbf{x}} = 0$ ) like it is the case for linear forcing ( $F_s = 0$ ) at integer  $t\omega/(2\pi)$ . Thus, it is not possible to “map” the state  $|\psi\rangle$  of an initially unforced system on  $|\psi_{\text{eff}}\rangle$  by suddenly switching on the forcing. However, one can smoothly switch on the drive. According to the adiabatic principle for quantum Floquet states [14],  $|\psi_{\text{eff}}\rangle$  can follow adiabatically when  $\hat{H}_{\text{eff}}$  is modified by the forcing, starting from  $|\psi_{\text{eff}}\rangle = |\psi\rangle$  in the undriven limit [9, 13]. Before passing from the ground state of the undriven system ( $J_{ij}^{\text{eff}} = J_{ij} < 0$ ) to the positive-hopping regime ( $J_{ij}^{\text{eff}} > 0$ ) in the presence of a trapping potential, the lattice should be tuned very deep, such that  $U \gg n_0|J|$  with filling  $n_0$  in the trap center. The system will form MI regions [15] with an integer number  $g$  of particles (depending on  $\mu_i/U$ ) localized at each site. Different MI regions will be separated only by tiny intermediate domains of non-integer filling. In the MI phases the state can follow smoothly through the sign-change of  $J$  when the lattice acceleration is ramped up continuously in a next step. Moreover, in a deep lattice unwanted interband transitions are strongly suppressed. When the desired strength of the forcing is reached, in the center of the trap the MI has to be melted. This can be achieved both by decreasing the lattice depth (without leaving the regime of strong correlation  $U \sim n_0|J|$ ) and by tuning the chemical potential in the trap center. The latter can be achieved by varying the trap, such that atoms are pushed into or pulled out of the center.

We have studied the MI-to-SF ( $J < 0$ ) and MI-to-Néel SF ( $J > 0$ ) transition in the triangular lattice theoretically. In the parameter plane spanned by  $\mu/U$  and  $J/U$ , a strong-coupling expansion as described in Ref. [16] gives the upper and lower boundary,  $\mu_p/U$  and  $\mu_h/U$ , of the MI phase with integer filling  $n = g$ . One finds  $\mu_p/U = g - (g+1)\eta - g c_p \eta^2 + \mathcal{O}(\eta^3)$  and  $\mu_h/U = (g-1) + g\eta + (g+1)c_h \eta^2 + \mathcal{O}(\eta^3)$ . The expansion parameter is given by  $\eta \equiv -\varepsilon(\mathbf{q})/U = w|J|/U$  with  $\varepsilon(\mathbf{q}) \equiv -|J|w$  being

the single-particle dispersion relation  $\varepsilon(\mathbf{p}) \equiv 2J[\cos(dp_x) + 2\alpha \cos(dp_x/2) \cos(\sqrt{3}dp_y/2)]$  evaluated at its minimum  $\mathbf{q}$ . The coefficients read  $c_p \equiv g + 1 - (5g+4)(1+2\alpha^2)/w^2$  and  $c_h \equiv g - (5g+1)(1+2\alpha^2)/w^2$ . Here  $w$  directly reflects frustration; while  $w = 4\alpha + 2$  for ferromagnetic  $J < 0$ , it is smaller for antiferromagnetic  $J > 0$ , namely  $w = \alpha^2 + 2$  for  $0 \leq \alpha \leq 2$  and  $w = 4\alpha - 2$  for  $\alpha \geq 2$ . As a consequence, the MI regions extend to larger values of  $|J|/U$  on the frustrated side of the  $J - \mu$  plane. This can also be observed in Fig. 3(b) displaying the phase diagram for  $\alpha = 1.3$ . Moreover, the transition from 1D like concave phase boundaries  $c_{p/h} < 0$  to square lattice like convex ones  $c_{p/h} > 0$  happens at noticeably larger  $\alpha$  in the case of frustration. Namely it occurs for  $\alpha$  between 0.03 and 0.13 (2.3 and 4.2) when  $J < 0$  (for  $J > 0$ ). For convex boundaries, also  $\mu_{p/h}/U \simeq \frac{1}{2}\{2g-1-\eta \pm [1-2(2g+1)\eta+\eta^2]^{1/2}\}$  obtained within meanfield approximation (cf. Refs. [15]) can be expected to provide a reasonable description.

The phase diagram plotted in Fig. 3(b) shows: the smaller  $n|J|/U$  gets, the smaller get the intervals of  $\mu/U$  with non-integer filling (i.e. outside the MI lobes). In order to reach the strong coupling limit  $U \gg n|J|$  at non-integer filling  $g < n < g+1$  [where the spin-1/2 description (2) with non-trivial polarization applies], the variation of  $\mu_i$  (i.e. of  $V_i$ ) must be smaller than  $\mu_h^{(g+1)} - \mu_p^{(g)} \sim 2(g+1)w|J|$  over an appreciable number of sites. Such a situation, where also the gapped SL phases are supposed to appear, can be achieved in the center of a shallow trap. Note that the presence of a (shallow) controllable trapping potential is definitely desirable: tuning its depth allows to manipulate the chemical potential/filling in the trap center. With respect to the chemical potential, the gapped SL phases, expected for large interaction and  $\alpha$  near 0.5 or 1.3 [cf. Fig. 2], would appear as incompressible regions at half-odd-integer filling. In Fig. 3(b) we have sketched these phases (shaded in grey); they show up as “bubbles” on the frustrated side between the MI regions.

**Experimental signatures of frustration.** – Experimental signatures of the Néel SF are sharp quasimomentum peaks at  $\mathbf{p} = \mathbf{q} + \text{reciprocal lattice vectors}$  [cf. Fig. 1(b)]. In the case of spiral order, the ordering vector  $\mathbf{q}$  can take two different values; when measuring (or already before) the system will spontaneously choose one of them. For a whole stack of 2D systems, the measurement will average over both quasimomentum distributions, unless there remains a finite coupling between the 2D-layers establishing the same order everywhere. Also the predicted downshift of the anisotropy ratio  $\alpha_0$  (where spiral continuously transforms into rhombic-staggered Néel order) with increasing interaction/lattice depth [cf. Figs. 2 and 3(a)] can be investigated experimentally. The growth of the staggered  $\alpha$ -domain with increasing quantum fluctuations is an example for “order by disorder” [1]. Another measurable consequence of frustration is the extension of the MI phases to larger values of  $|J|/U$  [cf. Fig. 3(b)]. Due to the lack of long-range order, the MI does not show sharp

peaks in the single- but rather in the two-particle momentum distribution (noise correlations) [17]. This is also true for the gapped SL phases, being the most striking implication of frustration expected. Thus, in order to distinguishing the SL from the MI experimentally, one should search for structures in the momentum distribution beyond sharp peaks. The SL can feature a pattern in the momentum distribution on the scale of a Brillouin zone (i.e. the inverse lattice spacing  $\pi/d$ ), reflecting delocalization of particles on pairs of neighboring sites forming “singlets”. Apart from that, single-site resolved measurements [18] clearly distinguish between MI and SL by number fluctuations.

**Conclusion and Outlook.** – We have proposed to realize the positive-hopping Bose-Hubbard model with a system of ultracold spinless atoms in a deep triangular optical lattice dressed by a rapid elliptical acceleration. Our scheme allows to experimentally investigate the physics of a frustrated quantum system under the clean and controlled conditions provided by ultracold atoms. Since frustration is induced to motional bosonic degrees of freedom, it is experimentally possible to reach temperatures that are low compared to the energy scales governing the system. The model smoothly approaches a quantum spin-1/2 XY-model in the deep-lattice limit of strong interaction. In order to draw the phase diagrams shown in Figs. 2 and 3(b) we have combined results from different approaches: (i) numerical simulations applying to the spin-1/2 limit of strong interaction at half-odd-integer filling (published recently in Ref. [5]), (ii) an above-Bogoliubov theory valid in the limit of weak interaction, and (iii) analytical strong-coupling as well as meanfield results for the limit of strong interaction at integer filling. Expected are superfluid phases showing staggered or spiral Néel order, Mott insulator phases having integer filling, and gapped spin-liquid phases at half-odd-integer filling. We have also described how the positive-hopping regime can be reached adiabatically, if initially the system is prepared in the usual negative-hopping ground state. Finally, experimental signatures of the different phases have been discussed. In conclusion, using an existing setup the experiment proposed here can provide novel information about a frustrated quantum system.

We have restricted our analysis to the triangular lattice geometry that we have implemented already in the laboratory. However, the route described here, namely (i) realizing a positive-hopping Bose-Hubbard model with ultracold atoms by dressing the system by a fast elliptical lattice acceleration and (ii) approaching the physics of a spin-1/2 XY-model in the limit of strong interaction, applies equally to other two-dimensional non-bipartite lattices such as the Kagomé lattice. This opens perspectives for interesting future research.

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## REFERENCES

- [1] C. LHUILLIER, arXiv:cond-mat/0502464 (2005); F. ALET, A.M. WALCZAK, AND M.P.A. FISHER, *Physica A*, **369** (2006) 369; R. MOESSNER AND A.P. RAMIREZ, *Physics Today*, **59-2** (2006) 24; S. SACHDEV, *Nature Phys.*, **4** (2008) 173.
- [2] C. NAYAK, S.H. SIMON, A. STERN, M. FREEDMAN, AND S.D. SARMA, *Rev. Mod. Phys.*, **80** (2008) 1083.
- [3] M. LEWENSTEIN, A. SANPERA, V. AHUFINGER, B. DAMSKI, A. SEN(DE), AND U. SEN, *Adv. Phys.*, **56** (2007) 243; I. BLOCH, J. DALIBARD, AND W. ZWERGER, *Rev. Mod. Phys.*, **80** (2008) 885.
- [4] B. DAMSKI, H.U. EVERTS, A. HONECKER, H. FEHRMANN, L. SANTOS, AND M. LEWENSTEIN, *Phys. Rev. Lett.*, **95** (2005) 060403.
- [5] R. SCHMIED, T. ROSCILDE, V. MURG, D. PORRAS, AND J. I. CIRAC, *New J. Phys.*, **10** (2008) 045017.
- [6] C. MORA AND Y. CASTIN, *Phys. Rev. A*, **67** (2003) 053615.
- [7] A. GRIFFIN, *Phys. Rev. B*, **53** (1996) 9341.
- [8] D.H. DUNLAP AND V.M. KENKRE, *Phys. Rev. B*, **34** (1986) 3625; F. GROSSMANN, T. DITTRICH, P. JUNG, AND P. HÄNGGI, *Phys. Rev. Lett.*, **67** (1991) 516.
- [9] A. ECKARDT, C. WEISS, AND M. HOLTHAUS, *Phys. Rev. Lett.*, **95** (2005) 260404.
- [10] H. LIGNIER, C. SIAS, D. CIAMPINI, Y. SINGH, A. ZENESINI, O. MORSCH, AND E. ARIMONDO, *Phys. Rev. Lett.*, **99** (2007) 220403.
- [11] A. ZENESINI, H. LIGNIER, D. CIAMPINI, O. MORSCH, AND E. ARIMONDO, *Phys. Rev. Lett.*, **102** (2009) 100403.
- [12] J.H. SHIRLEY, *Phys. Rev.*, **138** (1965) B979; H. SAMBE, *Phys. Rev. A*, **7** (1973) 6.
- [13] A. ECKARDT AND M. HOLTHAUS, *Phys. Rev. Lett.*, **101** (2008) 245302.
- [14] H.P. BREUER AND M. HOLTHAUS, *Phys. Lett. A*, **140** (1989) 507.
- [15] M.P.A. FISHER, P.B. WEICHMAN, G. GRINSTEIN, AND D.S. FISHER, *Phys. Rev. B*, **40** (1989) 546; S. SACHDEV, *Quantum Phase Transitions* (Cambridge University Press, Cambridge (UK)) 1999.
- [16] J.K. FREERICKS AND H. MONIEN, *EPL*, **24** (1994) 545.
- [17] E. ALTMAN, E. DEMLER, AND M.D. LUKIN, *Phys. Rev. A*, **70** (2004) 013603; R. BACH AND K. RZAŻEWSKI, *Phys. Rev. Lett.*, **92** (2004) 200401; S. FÖLLING, F. GERBIER, A. WIDERA, O. MANDEL, T. GERICKE, AND I. BLOCH, *Nature*, **434** (2005) 481.
- [18] T. GERICKE, P. WÜRTZ, D. REITZ, T. LANGEN, AND HERWIG OTT, *Nature Phys.*, **4** (2008) 949; W.S. BAKR, J.I. GILLEN, A. PENG, S. FOELLING, AND M. GREINER, *Nature*, **462** (2009) 74.